# 《微積分》

（一） 是非題 20%，(是，請在答案卡（A）欄劃記；非，請在答案卡（B）欄劃記。在其他欄位劃記者，不予計分，每題 2 分，答錯不倒扣。)

(B) 1. Given that $x = r\cos\theta$ and $y = r\sin\theta$, then $\frac{\partial x}{\partial r}\frac{\partial y}{\partial \theta} - \frac{\partial x}{\partial \theta}\frac{\partial y}{\partial r} = -r$.

(A) 2. If $f(x) = \frac{3x^2 + x}{x}$ and $0 < |x| < \varepsilon/3$ then $|f(x) - 1| < \varepsilon$.

(B) 3. If $\lim_{x\to\infty} \frac{f(x)}{g(x)} = -1$, then $\lim_{x\to\infty}[f(x) + g(x)] = 0$.

(A) 4. If $\sum a_n$ does not converge, then $\sum |a_n|$ does not converge.

(A) 5. If $f'$ is continuous on $R = (-\infty, \infty)$ and $\lim_{x\to\infty} f(x) = 0$, then $\int_1^\infty f'(x)dx = -f(1)$.

(B) 6. If $f$ is a differentiable real value function on $(a,b)$ and $c \in (a,b)$ such that $f'(c) = 0$, then $f$ has a relative maximum or minimum at *c*.

(B) 7. If $f'$ is bounded , then $f$ is bounded.

(B) 8. The $\lim_{x\to\infty}\left(x - \sqrt{x^2 + x}\right) = 0$.

(A) 9. If a function $f : [a,b] \to R$ is continuous, then $f$ is Riemann integrable.

(A) 10. $\int_{-2}^{3}\int_{1}^{5} e^{x^2-y}dxdy = \int_{1}^{5}\int_{-2}^{3} e^{x^2-y}dydx$.

（二） 選擇題：80 % (單選題，每題 5 分，答錯倒扣 1.25 分，倒扣至本大題零分爲止，未做答不給分亦不扣分。)

(D) 11. If the interval of convergence for power series $\sum_{n=0}^{\infty} a_n x^{n+1}$ is $(-2,2)$, then the interval of convergence for power series $\sum_{n=0}^{\infty} a_n (x-1)^{2n}$ is :

(A) $(-\sqrt{2}, \sqrt{2})$ (B) $(-2,2)$ (C) (0, 4)

(D) $(1-\sqrt{2},1+\sqrt{2})$ (E) $(-1,1)$

(C) 12. $\int_0^{\sqrt{3}} \frac{1}{(1+x^2)^{\frac{3}{2}}} dx = ?$

(A) $\frac{1}{2}$ (B) $\sqrt{3}$ (C) $\frac{\sqrt{3}}{2}$

(D) $-\frac{1}{2}$ (E) $-\frac{\sqrt{3}}{2}$

(E) 13. Find the arc length of the curve $y = \int_1^x \sqrt{t^5 - 1}\, dt$ , $1 \le x \le 4$.

(A) 36 (B) $\frac{30}{4}$ (C) $\frac{62}{5}$

(D) $\frac{126}{6}$ (E) $\frac{254}{7}$

(B) 14. Find the derivative of the function $f(x) = 2^{-x} \sin x^2$.

(A) $2^{-x+1} x \cos x^2 - 2^{-x-1} \sin x^2$

(B) $2^{-x+1} x \cos x^2 - 2^{-x} \ln 2 \sin x^2$

(C) $2^{-x} \cos x^2 - 2^{-x-1} \sin x^2$

(D) $2^{-x} x \cos x^2 - 2^{-x-1} \ln 10 \sin x^2$

(E) $2^{-x+1} x \cos x^2 - 2^{-x} \sin x^2$

(B) 15. $\int_4^6 \frac{1}{x^2 - 5x + 6} dx$ = ?

(A) 3 (B) $\ln 3 - \ln 2$ (C) $2 \ln 2$

(D) $\ln 2 - \ln 3$ (E) $2 \ln 3$

(D) 16. If $f(x) = \cos 2x$, find the value of $f^{(8)}(0)$.

(A) 128 (B) –128 (C) –256
(D) 256 (E) 512

(B) 17. The gamma function $\Gamma(n)$ is defined by $\Gamma(n) = \int_0^\infty x^{n-1} e^{-x} dx, n > 0$. Find $\Gamma(2)$.

(A) 0 (B) 1 (C) 2
(D) 3 (E) 4

(D) 18. Find the area of the region that lies inside both the curves $r = 4\sin\theta$ and $r = 4\cos\theta$.

(A) $2\pi + 1$ (B) $2\pi - 1$ (C) $2\pi + 4$
(D) $2\pi - 4$ (E) $2\pi$

(C) 19. Find the terms of the Maclaurin series for $f(x) = \frac{1}{\sqrt{1+2x}}$, as far as the term in $x^3$.

(A) $1 - x + x^2 - x^3$

(B) $1 + x - \frac{1}{2}x^2 + \frac{1}{3}x^3$

(C) $1 - x + \frac{3}{2}x^2 - \frac{5}{2}x^3$

(D) $1 + x + 3x^2 + 5x^3$

(E) $1 - x + \frac{3}{2}x^2 - \frac{7}{3}x^3$

(B) 20. Find the volume of the solid bounded below by the *xy*-plane and above by the paraboloid $z = 1 - (x^2 + y^2)$.

(A) π (B) $\frac{\pi}{2}$ (C) $\frac{\pi}{3}$
(D) $\frac{\pi}{4}$ (E) $\frac{\pi}{8}$

(C) 21. How many local maximum points in the following function:

$f(x, y) = -\frac{1}{4}x^4 + \frac{2}{3}x^3 + 4xy - y^2$.

(A) none (B) 1 (C) 2
(D) 3 (E) 4

(E) 22. $\lim_{n\to\infty}\left(\frac{n}{n^2+1} + \frac{n}{n^2+4} + \frac{n}{n^2+9} + \cdots + \frac{n}{2n^2}\right) = ?$

(A) 0 (B) $\frac{1}{2}$ (C) $\pi$
(D) $\frac{\pi}{2}$ (E) $\frac{\pi}{4}$

(C) 23. If $F(x) = \int_0^{x^2} \frac{\sin(xt)}{t} dt$, find $F'(x)$ where $x \neq 0$.

(A) $\frac{3\sin x^3}{x^2}$ (B) $\frac{\sin x^3}{x^2}$ (C) $\frac{3\sin x^3}{x}$

(D) $\frac{2\cos x^3}{x}$ (E) $\frac{2\sin x^3}{x}$

(E) 24. $\lim_{x\to 0}\frac{3\sin\pi x-\sin 3\pi x}{x^3}=?$

(A) 0 (B) $\pi$ (C) $\pi^2$

(D) $2\pi^3$ (E) $4\pi^3$

(A) 25. $\int_0^4\int_{y/2}^{2} e^{x^2}\,dxdy$ = ?

(A) $e^4-1$ (B) $e^4$ (C) $\frac{e^4-1}{4}$

(D) $\frac{e^2-1}{2}$ (E) $\frac{e^2}{2}$

(C) 26. The maximum value of the function $f(x,y,z)=x+2y+3z$ subject to the constraint $x^2+y^2+z^2=25$ is _____.

(A) $7\sqrt{14}$ (B) $\frac{60}{\sqrt{14}}$ (C) $5\sqrt{14}$

(D) 15 (E) $\frac{30}{\sqrt{14}}$