高雄醫學大學九十一學年度學士後西醫學系招生考試試題

科目：微積分

壹、是非題20%（每題2分，答錯不倒扣）

(╳) 1.A function f has an extremum at a number c when $f'(c)=0$

理由： $f'_{(c)}$ **可能不存在**

(╳) 2. $\int_{-2}^{1}\frac{1}{x^4}dx=-\frac{3}{8}$

$\int_{-2}^{1}\frac{1}{X^4}dx=\int_{-2}^{0}\frac{1}{X^4}dx+\int_{0}^{1}\frac{1}{X^4}dx$ **（兩個瑕積分均不存在）**

(○) 3.If f has a local minimum at (a,b) an f is differentiable at (a,b), then $\nabla f(a,b)=0$

(╳) 4.Suppose that $\lim_{n\to\infty} f(\frac{1}{n})=L$, $n\in N$, then $\lim_{n\to 0+} f(x)=L$

例如： $f(x)=\sin\frac{p}{x}$

$\lim_{n\to\infty} f(\frac{1}{n})=\lim_{n\to\infty}\sin\frac{p}{\frac{1}{n}}=\lim_{n\to\infty}\sin np=\lim_{n\to\infty}0=0$

但 $\lim_{x\to 0^+} f(x)=\lim_{x\to 0^+}\frac{p}{x}$ **呈振盪不存在**

(○) 5.Let $f(x)=\begin{cases}\sin x/x & x\neq 0\\ 1 & x=0\end{cases}$. Define $F(x)=\int_0^x f(t)dt.$ Then F is differentiable for all x in R.

理由： $f(x)e$ cont on R ∴**滿足微積分基本定理**

(○) 6.There exist x∈[0,1] such that $e^x=\int_0^1 e^{t^2}dt.$

(○) 7.Let f be a two variables real function. If f is differentiable at (x_0,y_0), then f is continuous at (x_0,y_0).

(○) 8.If f and g are continuous functions in [a,b], and $g(x)\ge 0$ in [a,b], then there exists a number c in [a,b] such that $\int_a^b f(x)g(x)dx=f(c)\int_a^b g(x)dx$。

(╳) 9.If series $\sum_{n=1}^{\infty}a_n$ and $\sum_{n=1}^{\infty}(-a_n)$ are convergent, then $\sum_{n=1}^{\infty}|a_n|$ is also convergent.

則 $\sum a_n$ **與** $\sum(-a_n)$ **均收斂**

但 $\sum|a_n|$ **卻發散**

(╳)10.An integrable function is always continuous.**只要** piecewisely continuous **就可積分**

貳、選擇題80%（單選題，每題答對得5分，答錯倒扣1.25分）

(A) 11.If $x\sin(px)=\int_0^{x^2} f(t)dt$, where f is a continuous function, then f (4)=

(A) $\frac{\pi}{2}$ (B) $\frac{1}{2}$ (C) $\frac{\pi}{4}$ (D) $\frac{1}{4}$ (E) 2π

解： $X\sin(\pi^{x}) = \int_0^{x^2} f(t)dt$

兩邊對 X 微分

$\sin(\pi x) + x\cos(\pi x)\cdot\pi = f(x^2)\cdot 2x$

令 $x^2 = 4 \Rightarrow x = -2$ **或** 2

1.$x = 2$

$\sin(2\pi) + 2\cos(2\pi)\cdot\pi = f(4)\cdot 2\cdot 2$

$\Rightarrow 0 + 2\cdot 1\cdot\pi = f(4)\cdot 4$

$\Rightarrow f(x) = \frac{\pi}{2}$

2.$x = -2$

$\sin(-2\pi) - 2\cos(-2\pi)\cdot\pi = f(4)\cdot 2\cdot(-2)$

$\Rightarrow 0 - 2\cdot 1\cdot\pi = f(4)\cdot(-4)$

$\Rightarrow f(x) = \frac{\pi}{2}$

(B) 12.For what value of *a* is it true that $\lim_{x\to\infty}\left(\frac{x+a}{x-a}\right)^{x} = e$

(A) $a = \frac{e}{2}$ (B) $a = \frac{1}{2}$ (C) $a = -\frac{1}{2}$ (D) $a = -2$ (E) $a = \frac{2}{e}$

解： $\lim_{x\to\infty}\left(\frac{x+a}{x-a}\right)^{x} = e$

$\lim_{x\to\infty}\left(\frac{x+a}{x-a}\right)^{x}$ $(1^{+\infty}$ **型**$)$

$= \exp[\lim_{x\to\infty} x\cdot\ln\frac{x+a}{x-a}](\infty\cdot 0$**型**$)$

$= \exp[\lim_{x\to\infty}\frac{\ln\frac{1+\frac{a}{x}}{1-\frac{a}{x}}}{\frac{1}{x}}](\frac{0}{0}$**型**$)$

$y\to 0^{+}\ \frac{\ln(1+ay-\ln 1-ay)}{y}$ (**令** $y = \frac{1}{x}$)

<u>L' HR</u> $\exp[\lim_{y\to 0+}\frac{\frac{a}{1+ay} - \frac{-a}{1-ay}}{1}]$

$\exp[2a = e^{2a}] = e$

$\therefore 2a = 1 \Rightarrow a = \frac{1}{2}$

(D) 13.The area of the region $S = \{(x,y): x\ge 0, y\le 1, x^2+y^2\le 4y\}$ is

(A) $\frac{2\pi}{3}+\frac{\sqrt{3}}{2}$ (B) $\frac{2\pi}{3}+\frac{1}{2}-\sqrt{3}$ (C) $\frac{4\pi}{3}-\sqrt{3}$ (D) $\frac{2\pi}{3}-\frac{\sqrt{3}}{2}$ (E) $\frac{4\pi}{3}-\frac{\sqrt{3}}{2}$

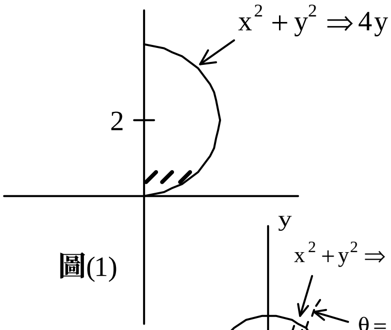
解：所求區域（如圖 1 斜線部份）

面積 A 與（圖 2 敘線部份面積）同

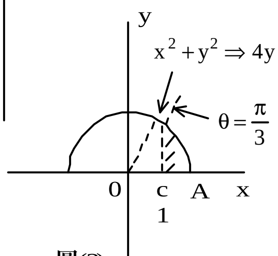
A =（**扇形** OAC **面積**）–（**三角形** OBC **面積**）

$$=\frac{1}{2}(\frac{\pi}{3})\cdot(2^2)-\frac{1}{2}1\cdot\sqrt{3}$$

$$=\frac{2}{3}\pi-\frac{\sqrt{3}}{2}$$



圖(1)



圖(2)

另解：

由圖形(1)

$$A=\iint dA$$

$$=\int_0^1[\int_0^{\sqrt{4y-y^2}}dx]dy$$

$$=\int_0^1\sqrt{4y-y^2}dy$$

$$=\frac{2}{3}\pi-\frac{\sqrt{3}}{2}$$

(E) 14.The average value of the function $f(x)=\int_x^1\cos(t^2)dt$ on the interval [0,1] is

(A)sin1 (B) $\frac{\pi\sin 1}{2}$ (C) $\frac{\cos 1}{4}$ (D) $\frac{\pi}{4}$ (E) $\frac{\sin 1}{2}$

解： $f(x)=\int_x^1\cos(t^2dt)$

$$<f(x)>=\frac{1}{1-0}\int_0^1 f(x)dx$$

$$=\int_0^1[\int_x^1\cos(t^2)dt]dx$$

$$=\int_0^1[\int_0^t\cos(t^2)dx]dx$$

$$=\int_0^1 x\cos(t^2)\Big|_0^t dt$$

$$=\int_0^1 t\cos(t^2)dt$$

$$=\frac{1}{2}\sin(t^2)\Big|_0^1=\frac{1}{2}\sin 1$$

(C) 15. $\int_{-3}^{3}\int_0^{\sqrt{9-x^2}}\sqrt{x^2+y^2}\,dydx=$

(A)3π (B)6π (C)9π (D)12π (E)18π

解：令 $I=\int_{-3}^{3}[\int_0^{\sqrt{9-x^2}}\sqrt{x^2+y^2}dy]dx$

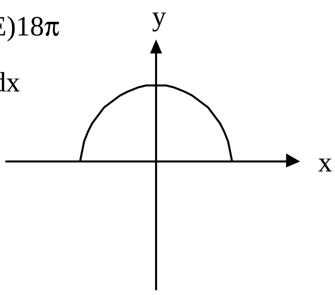
令 $x=\cos\theta$**，** $y=r\sin\theta$

$(r\in[0,\infty),\theta\in[0,2\pi))$

則 $I=\int_0^{\pi}[\int_0^3 r\cdot rdr]d\theta$

$$=[\int_0^{\pi}d\theta]\cdot[\int_0^3 r^2dr]$$

$$=\pi\cdot\frac{1}{3}r^3\Big|_0^3=9\pi$$



(E) 16.Suppose that $z = f(x, y)$ is differentiable such that $\frac{\partial z}{\partial x} = 3x^2 - 3y$, $\frac{\partial z}{\partial y} = -3x + y$, and $f(0,0) = 2$, then $f(1,2) =$

(A)11 (B)−3 (C)$-\frac{5}{2}$ (D)$\frac{3}{2}$ (E)−1

解：

$$\begin{cases} \frac{\partial z}{\partial x} = 3x^2 - 3y \cdots\cdots (1) \\ \frac{\partial z}{\partial y} = -3x + y \cdots\cdots (2) \end{cases}$$

(1) $z = x^3 - 3xy + g(y) \cdots\cdots (3)$

(3)**代入**

$$-3x + g(y) = -3x + y$$

$$\Rightarrow g_{(y)} = y$$

$$\Rightarrow g(y) = \frac{1}{2}y^2 + c$$

$$\therefore f(x, y) = x^3 - 3xy + \frac{1}{2}y^2 + c$$

$$\therefore f(0,0) = 2 \Rightarrow c = 2$$

$$\therefore f(x, y) = x^3 - 3xy + \frac{1}{2}y^2 + 2$$

$$\therefore f(1,2) = 1^3 - 3 \cdot 1 \cdot 2 + \frac{1}{2} \cdot 2^2 + 2$$

$$= -1$$

(D) 17.Consider the power series $\sum_{n=1}^{\infty} \frac{x^n}{n}$. The interval of convergence is

(A)$(-\infty,\infty)$ (B)$[-1,1]$ (C)$(-1,1\ ]$ (D)$[-1,1\ )$ (E)$(-1,1)$

解： $\sum_{n=1}^{\infty} \frac{x^n}{n}$

根據根值審斂法，級數收斂充分條件：

$$\lim_{n \to \infty} \sqrt[n]{\left| \frac{x^n}{n} \right|} < 1$$

$$\Rightarrow |x| < 1 \Rightarrow -1 < x < 1$$

1° $x = 1$, **級數呈** $\sum \frac{1}{n}$ div

2° $x = -1$, **級數呈** $\sum \frac{(-1)^n}{n}$ conv

收斂區間： $[-1,1]$

(D) 18.Evaluate the limit. $\lim_{h \to 0} \left\{ \frac{\int_0^{(1+h)^2} e^{x^2} dx - \int_0^1 e^{x^2} dx}{h} \right\} =$

(A)1 (B)2 (C)e (D)2e (E)Does not exist.

解： $\lim_{h \to 0} \frac{\int_0^{(1+h)^2} e^{x^2} dx - \int_0^1 e^{x^2} dx}{h}$

$\underline{\underline{\text{L' HR}}}$ $\lim_{h \to 0} e^{(1+h)^4} \cdot 2(1+h) = 2 \cdot \ell = 2\ell$

(B) 19. $\int_{1}^{\sqrt{3}} \frac{x-1}{x^3+x}dx =$

(A) $\frac{1}{2}\ln 6+\frac{\pi}{12}$ (B) $\frac{1}{2}\ln\frac{2}{3}+\frac{\pi}{12}$ (C) $-\frac{1}{2}\ln 6-\frac{\pi}{12}$ (D) $-\frac{1}{2}\ln\frac{2}{3}-\frac{\pi}{12}$ (E) $-\frac{1}{2}\ln\frac{2}{3}+\frac{\pi}{12}$

解： $\int_{1}^{\sqrt{3}} \frac{x-1}{x^3+x}dx$

$$\int \frac{x-1}{x^3+x}dx = \int \frac{x-1}{x(x^2+1)}dx$$

$$= \int [\frac{-1}{x}+\frac{x+1}{x^2+1}]dx$$

$$= (-1)\ln X + \frac{1}{2}\ln(x^2+1) + \tan^{-1} X$$

$$\therefore \textbf{原式} = [(-1)\ln x\sqrt{3} + \frac{1}{2}\ln(x^2+1) + \tan^{-1} X]\Big|_{1}^{\sqrt{3}}$$

$$= [(-1)\ln\sqrt{3} + \frac{1}{2}\ln 4 + \tan^{-1}\sqrt{3}]$$

$$= [(-1)\ln 1 + \frac{1}{2}\ln 2 + \tan^{-1} 1]$$

$$= -\ln\sqrt{3} + \ln 2 + \frac{\pi}{3} - \frac{1}{2}\ln 2 - \frac{\pi}{4}$$

$$= -\ln\sqrt{3} + \frac{1}{2}\ln 2 + \frac{\pi}{12}$$

$$= \frac{\pi}{12} + \frac{1}{2}\ln\frac{2}{3}$$

(B) 20.Let $f(x) = \begin{cases} x\sin\frac{1}{x} & x \neq 0 \\ 0 & x = 0 \end{cases}$. Which of the following is incorrect?

(A)Function f is continuous at x = 0

(B)Function f is differentiable at x = 0

(C)Graph of f has a horizontal asymptote y = 1

(D)Function f is continuous at x ≠ 0

(E) Function f is concave down for $x > \frac{6}{\pi}$.

解： $f(x) = \begin{cases} x\sin\frac{1}{x} & x \neq 0 \\ 0 & x = 0 \end{cases}$

$$f'_{(0)} = \lim_{x\to 0}\frac{f(x)-f(0)}{x-0}$$

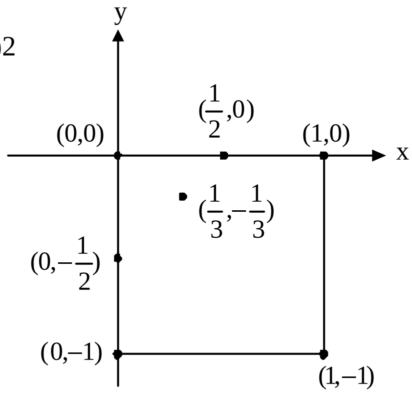
$$= \lim_{x\to 0}\frac{x\sin\frac{1}{x}-0}{x-0}$$

$$= \lim_{x\to 0}\sin\frac{1}{x} \quad (\textbf{不存在})$$

$$\therefore f\textbf{在}x=0\textbf{不可微}$$

(D) 21.Let $f(x,y)=x^2-xy+y^2-x+y$. The absolute maximum of f on rectangle $R=\{(x,y)|0\le x\le 1,-1\le y\le 0\}$ is

(A) $-\frac{1}{3}$ (B) $-\frac{1}{4}$ (C)0 (D)1 (E)2



解： $f(x,y)=x^2-xy+y^2-x+y$

1° $0<x<1,\ -1<y<0$

$$\begin{cases} fx(x,y)=2x-y-1=0 \\ fy(x,y)=-x+2y+1=0 \end{cases}$$

$\Rightarrow (x,y)=(\frac{1}{3},-\frac{1}{3})$

2° $x=0\quad -1\le y\le 0$

令

$f(x,y)=y^2+y=g(y)$

$g'_{(y)}=2y+1=0\Rightarrow y=-\frac{1}{2}$

$(x,y)=(0,-\frac{1}{2})$

3° $x=1,-1\le y\le 0$

$f(x,y)=1-y+y^2-1+y=y^2=g(y)$

$g'_{(y)}=0\Rightarrow y=0$

$(x,y)=(1,0)$

4° $y=0,0\le x\le 1$

$f(x,y)=x^2-x=g(x)$

$g'_{(x)}=2x-1=0\Rightarrow x=\frac{1}{2}$

$(x,y)=(\frac{1}{2},0)$

5° $y=-1,0\le x\le 1$

令

$f(x,y)=x^2+x+1-x-1=x^2=g(x)$

$g'(x)=0\Rightarrow x=0$

$f(\frac{1}{3},-\frac{1}{3})=-\frac{1}{3}$

$f(0,-\frac{1}{2})=-\frac{1}{4}$

$f(1,0)=0$

$f(\frac{1}{2},0)=-\frac{1}{2}$

$f(0,-1)=0$

$f(1,-1)=1$

$f(0,0)=0$

$\therefore$ **得最大值**$:1$

(A) 22.The maximum directional derivative of $f(x,y)=x^2-xy-\frac{5}{2}y^2$ at point (1,–1) is

(A)5 (B)10 (C)15 (D)20 (E)25

解： $f(x,y)=x^2-xy-\frac{5}{2}y^2$

$\nabla f(x,y)=[2x-y,-x-5y]$

$\nabla f(1,-1)=[3,4]$

$\therefore \|\nabla f(1,-1)\|=\sqrt{3^2+4^2}$ **（最大變化率）**

$=5$

(C) 23.If f is a continuous function in [0,2], then $\int_0^2 \frac{f(x)}{f(x)+f(2-x)}dx=$

(A)0 (B) $\frac{1}{2}$ (C)1 (D) $\frac{3}{2}$ (E)2

解：令 $I=\int_0^2 \frac{f(x)}{f(x)-f(2-x)}dx$

並令 $x=2-y \Rightarrow dx=-dy$

$I=\int_2^0 \frac{f(2-y)}{f(2-y)-f(y)}(-dy)$

$=\int_0^2 \frac{f(2-y)}{f(2-y)-f(y)}dy=\int_0^2 \frac{f(2-x)}{f(2-x)-f(x)}dx$

$\Rightarrow I+I=\int_0^2 \frac{f(x)}{f(x)-f(2-x)}dx+\int_0^2 \frac{f(2-x)}{f(2-x)-f(x)}dx$

$=\int_0^2 \frac{f(x)-f(2-x)}{f(x)-f(2-x)}dx=\int_0^2 1dx=2$

$\therefore I=1$

(B) 24.The Fibonacci sequence $\{f_n\}$ was defined by $f_1=1, f_2=1, f_n=f_{n-1}+f_{n-2}$ for $n\geq 3$. Then $\sum_{n=2}^{\infty}\frac{1}{f_{n-1}f_{n+1}}=$

(A)0 (B)1 (C)2 (D)$\frac{5}{2}$ (E)$\frac{10}{3}$

解：令 $fn=r^n$**，由差分方程可得**

$r^2=r+1$

$\Rightarrow r^2-r-1=0$

$\Rightarrow r=\frac{1+\sqrt{5}}{2}$ **或** $r=\frac{1-\sqrt{5}}{2}$

$\therefore f_n=A(\frac{1+\sqrt{5}}{2})^n+B(\frac{1-\sqrt{5}}{2})^n$

而 $f_1=1, f_2=2$**，易得**

$A=\frac{1}{\sqrt{5}}, B=-\frac{1}{\sqrt{5}}$

i.e. $fn=\frac{1}{\sqrt{5}}[(\frac{1+\sqrt{5}}{2})^n-(\frac{1-\sqrt{5}}{2})^n]$

$\Rightarrow \frac{1}{f_{n-1}\cdot f_{n+1}}=\frac{1}{f_{n-1}f_n}-\frac{1}{fn\cdot f_{n+1}}$

$\therefore \sum_{n=2}\frac{1}{f_{n-1}\cdot f_{n+1}}=\sum_{n=2}^{\infty}[\frac{1}{f_{n-1}\cdot f_n}-\frac{1}{f_n\cdot f_{n+1}}]$

$=\frac{1}{f_{2-1}\cdot f_2}-\lim_{n\to\infty}\frac{1}{f_{n-1}\cdot f_n}$ (telescopin g**級數性質**)

$=\frac{1}{f_1\cdot f_2}=\frac{1}{I\cdot 1}$

(C) 25.The volume of the solids obtained by rotating the region bounded by the curves $y = x$ and $y = x^2$ about the line y = 2 is

(A) $\frac{2\pi}{15}$ (B) $\frac{\pi}{6}$ (C) $\frac{8\pi}{15}$ (D) $\frac{3\pi}{2}$ (E) 2π

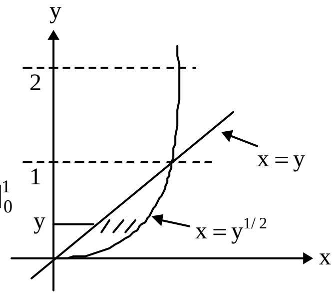
解： $V = \int_0^1 2\pi(2-y)(y^{1/2} - y)dy$

$= 2\pi\int_0^1 (2y^{1-2} - 2y - y^{3-2} + y^2)dy$

$= 2\pi \cdot [2 \cdot \frac{2}{3}y^{3/2} - y^2 - \frac{2}{5}y^{5/2} + \frac{1}{3}y^3]_0^1$

$= 2\pi[\frac{4}{3} - 1 - \frac{2}{5} + \frac{1}{3}]$

$= 2\pi\frac{20-15-6+5}{15} = \frac{8\pi}{15}$



(B) 26. $D(\sec^{-1}\sqrt{x}) =$

(A) $\frac{1}{x\sqrt{x^2-1}}$ (B) $\frac{1}{2x\sqrt{x-1}}$ (C) $\frac{\sec^{-1}\sqrt{x}\tan^{-1}\sqrt{x}}{2\sqrt{x}}$ (D) $\frac{\sec\sqrt{x}\tan\sqrt{x}}{2\sqrt{x}}$ (E) $\frac{-\tan\sqrt{x}}{2\sqrt{x}}$

解： $D(\sec^{-1}\sqrt{x}) = \frac{1}{\sqrt{x}\sqrt{(\sqrt{x})^2 - 1}} \cdot \frac{1}{2\sqrt{x}}$

$= \frac{1}{2x\sqrt{x-1}}$