

高雄醫學大學九十二學年度學士後醫學系招生考試試題

科目:微積分

考試時間: 80 分鐘

共 三 頁

說明:一.選擇題用 2B 鉛筆在「答案卡」上作答,修正時應以橡皮擦拭,切勿使用修正液(帶),未遵照正確作答方法而致無法判讀者,考生自行負責。  
二.試卷必須繳回,不得攜出試場。

(一) 是非題： 20%。(是,請在答案卡(A)欄位劃記;非,請在答案卡(B)欄位劃記。在其它欄位劃記者,不予計分。每題 2 分,答錯不倒扣。)

1. If  $f(x) > 1$  for all  $x$  and  $\lim_{x \rightarrow 0} f(x)$  exists, then  $\lim_{x \rightarrow 0} f(x) > 1$ .
2. If  $\sum a_n$  is divergent, then  $\sum |a_n|$  is divergent.
3.  $\int_{-1}^1 \int_0^1 e^{x^2+y^2} \sin y \, dx \, dy = 0$ .
4. Let  $f : [a, b] \rightarrow [a, b]$  be a continuous function, then there exists  $x \in [a, b]$  such that  $f(x) = x$ .
5. If  $\lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}$  doesn't exist, then  $\lim_{x \rightarrow a} \frac{f(x)}{g(x)}$  doesn't exist too.
6.  $\frac{b-a}{1+b^2} < \tan^{-1} b - \tan^{-1} a < \frac{b-a}{1+a^2}$  for  $0 \leq a < b$ .
7. Suppose  $f$  is integrable on  $[a, b]$ , define  $F(x) = \int_a^x f(t) dt$ , then  $F$  is differentiable on  $(a, b)$ .
8. Let  $f$  be a continuous function defined on a closed interval  $[1, 3]$  and  $f(x) \leq 3$  for all  $x \in [1, 3]$ . Define  $F(x) = \int_1^x t^2 f(t) dt$  for  $x \in [1, 3]$ , then  $F(3) \leq 26$ .
9. Let  $f$  be a function defined on the set  $D = \{(x, y) \mid -1 \leq x \leq 1, -1 \leq y \leq 1\}$ . If  $\frac{\partial f}{\partial x}(0,0)$  and  $\frac{\partial f}{\partial y}(0,0)$  exist, then  $f$  is differentiable at  $(0,0)$ .
10. Let  $f$  be a continuous function defined on the bounded interval  $(a, b)$ , then there exist a point  $x_0 \in (a, b)$  such that  $f(x) \leq f(x_0)$  for all  $x \in (a, b)$ .

(二) 選擇題： 80%。(單選題,每題 5 分,答錯一題倒扣 1.25 分,倒扣至本大題零分為止,未作答者不給分亦不扣分。)

11.  $\int_{-1}^3 \frac{6x-7}{3x+5} dx =$  \_\_\_\_\_.  
 (A)  $8 - \frac{17}{3} \ln 14$       (B)  $8 - \frac{17}{3} \ln 7$       (C)  $8 - \frac{\ln 17}{3}$       (D)  $4 - \frac{17}{3} \ln 14$       (E)  $4 - \frac{17}{3} \ln 7$
12. The area between the curve  $y = x\sqrt{3x+1}$  and the lines  $y = 0$ ,  $x = 0$  and  $x = 1$  is \_\_\_\_\_.  
 (A)  $\frac{116}{135}$       (B)  $\frac{116}{125}$       (C)  $\frac{106}{135}$       (D)  $\frac{4}{135}$       (E)  $\frac{4}{125}$

13. Find the equation of the curve that satisfies the differential equation  $yy' + 2x = 0$  and that passes through the point  $(3, -1)$ .
- (A)  $-x^2 + 9 = \ln|y|$  (B)  $\frac{y^2}{2} = -x^2 + \frac{11}{2}$  (C)  $y^2 + 2x^2 = 19$  (D)  $2x^2 - y^2 = 18$  (E)  $y^2 - 2x^2 + 17 = 0$
14. If  $f(u, v, w)$  is differentiable and  $u = x - y$ ,  $v = y - z$  and  $w = z - x$ , then  $\frac{\partial f}{\partial x} + \frac{\partial f}{\partial y} + \frac{\partial f}{\partial z} = ?$
- (A)  $-3$  (B)  $0$  (C)  $3$  (D)  $-\frac{\partial f}{\partial u} + \frac{\partial f}{\partial v} - \frac{\partial f}{\partial w}$  (E)  $x \frac{\partial f}{\partial x} + y \frac{\partial f}{\partial y} + z \frac{\partial f}{\partial z}$
15. If  $f(x, y) = xe^y$ , then the rate of change of  $f$  at the point  $P(2, 0)$  in the direction from  $P$  to  $Q(\frac{1}{2}, 2)$  is:
- (A)  $-\frac{11}{2}$  (B)  $-\frac{5}{2}$  (C)  $1$  (D)  $\frac{5}{2}$  (E)  $\frac{11}{2}$
16. Let  $f(x) = \left[ \frac{(x+1)^4(x-5)^2}{x-1} \right]^{\frac{1}{3}}$ , then  $f'(2) = \underline{\hspace{2cm}}$ .
- (A)  $1$  (B)  $2$  (C)  $3$  (D)  $-1$  (E)  $-2$
17. Suppose  $a, b > 0$ , then  $\lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{1}{an + bk} = \underline{\hspace{2cm}}$ .
- (A)  $\frac{1}{a} \ln \frac{a+b}{a}$  (B)  $\frac{1}{b} \ln \frac{a+b}{a}$  (C)  $\frac{1}{a} \ln \frac{a+b}{b}$  (D)  $\frac{1}{b} \ln \frac{a+b}{b}$  (E)  $\ln \frac{b}{a}$
18.  $\int_0^{\frac{\pi}{3}} |\sin x - \cos x| dx = \underline{\hspace{2cm}}$
- (A)  $\frac{\sqrt{3}-1}{2}$  (B)  $\frac{\sqrt{3}+1}{2}$  (C)  $2\sqrt{2} - \frac{3+\sqrt{3}}{2}$  (D)  $2\sqrt{2} - \frac{1+\sqrt{3}}{2}$  (E)  $\frac{1-\sqrt{2}+\sqrt{3}}{2}$
19. Let  $f(x) = xe^{-x}$  and  $p(x) = a_0 + a_1x + a_2x^2 + \dots + a_nx^n + \dots$  be the Maclaurin series of  $f(x)$ , then  $a_4 = \underline{\hspace{2cm}}$ .
- (A)  $\frac{1}{3!}$  (B)  $\frac{-1}{3!}$  (C)  $\frac{1}{4!}$  (D)  $\frac{-1}{4!}$  (E)  $\frac{1}{5!}$
20. The slope of the tangent line to the polar curve  $r = \frac{\sqrt{2}}{2} + \cos \theta$  at the point  $(r, \theta) = (\sqrt{2}, \frac{\pi}{4})$  is:
- (A)  $-1$  (B)  $-\frac{1}{2}$  (C)  $-\frac{1}{3}$  (D)  $-\frac{2}{3}$  (E)  $-\frac{3}{2}$
21. The volume of the solid bounded by  $z = 2 - x^2 - y^2$  and  $z = x^2 + y^2$  is:
- (A)  $\frac{2}{3}\pi$  (B)  $\frac{3}{2}\pi$  (C)  $\frac{3}{4}\pi$  (D)  $\frac{4}{3}\pi$  (E)  $\pi$
22. Let  $\frac{\sin 2x}{x} \leq f(x) \leq \frac{e^{2x}-1}{x}$  for  $x \in (0, 0.5)$ , then  $\lim_{x \rightarrow 0^+} (2f(x))^{f(x)} = \underline{\hspace{2cm}}$ .
- (A)  $1$  (B)  $2$  (C)  $4$  (D)  $16$  (E)  $e^1$
23. Define the function  $f$  by  $f(x) = \int_0^x (t-t^3)e^t dt$ . Which of the following statement is correct?
- (A) Function  $f$  derives its absolute maximum at point  $x = -1$   
 (B) Function  $f$  derives its absolute maximum at point  $x = 0$   
 (C) Function  $f$  derives its absolute maximum at point  $x = 1$   
 (D) Function  $f$  derives its absolute minimum at point  $x = -1$   
 (E) Function  $f$  does not have absolute maximum or minimum value

24. Define  $f(x) = \int_0^x (\cos t)^4 dt$ . Which of the following statement is *false*?

- (A)  $f$  is a strictly increasing function
- (B)  $f'(x) = (\cos x)^4$
- (C)  $f(x+2\pi) - f(x)$  is constant
- (D)  $f(x) \geq 0$  for all real number  $x$
- (E)  $f(0) = 0$

25. Let  $D = \{(x, y) \mid 0 \leq x \leq \frac{\pi}{2}, 1 \leq y \leq 2\}$ , then  $\iint_D x \cos(xy) dA =$  \_\_\_\_\_.

- (A)  $-\frac{\pi}{2}$
- (B)  $-1$
- (C)  $0$
- (D)  $\frac{\pi}{2}$
- (E)  $1$

26.  $\int_1^e x(\ln x)^2 dx =$  \_\_\_\_\_.

- (A)  $-\frac{1}{4}(e^2 + 1)$
- (B)  $\frac{1}{4}(e^2 - 1)$
- (C)  $\frac{1}{4}(1 - e^2)$
- (D)  $\frac{1}{4}(e^2 + 1)$
- (E)  $-\frac{1}{4}(e + 1)$

高雄醫學大學九十二年  
學年度招生委員會

## 高雄醫學大學 92 年度學士後西醫招生考試試題詳解

## 科目：微積分

徐中老師解題

1. (B)

理由：(舉例說明)

$$f(x) = \begin{cases} \frac{x^2+1}{x^4+1} & 0 < |x| < 1 \\ 2 & \text{otherwise} \end{cases}$$

則  $f(x) > 1$  for all  $x$ 

$$\text{而 } \lim_{x \rightarrow 0} f(x) = \lim_{x \rightarrow 0} \frac{x^2+1}{x^4+1} = 1$$

2. (A)

理由：(證明如下)

$$\because \sum |a_n| \text{ conv.} \Rightarrow \sum a_n \text{ conv.}$$

其對偶命題為真：

$$\sum |a_n| \text{ div} \Leftrightarrow \sum a_n \text{ div}$$

3. (A)

理由：(證明如下)

$$\begin{aligned} & \int_{-1}^1 \int_0^1 e^{x^2+y^2} \sin y \, dx \, dy \\ &= \int_{-1}^1 \left[ \int_0^1 e^{x^2+y^2} \sin y \, dx \right] dy = \left[ \int_{-1}^1 e^{y^2} \sin y \right] \cdot \left[ \int_0^1 e^{x^2} dx \right] \\ &= 0 \quad \left( \because e^{y^2} \sin y \text{ 為奇函數}, \therefore \int_{-1}^1 e^{y^2} \sin y \, dy = 0 \right) \end{aligned}$$

4. (A)

理由：(證明如下)

(情況一)

$$f(a) \neq a \text{ 且 } f(b) \neq b$$

$$\text{令 } g(t) = f(t) - t$$

$$\text{則 } g(t) \in \text{cont. on } [a, b]$$

$$g(a) = f(a) - a > 0 \quad (\because f(a) \in [a, b], \text{ 且 } f(a) \neq a)$$

$$g(b) = f(b) - b < 0 \quad (\because f(b) \in [a, b], \text{ 且 } f(b) \neq b)$$

$$\therefore g(a) \cdot g(b) < 0$$

由中值定理可得：

$f(a) = a$  或  $f(b) = b$   
則存在  $a \in [a, b]$   
使得  $f(a) = a$  或

(情況二)

$$\exists x \in (a, b) \subset [a, b]$$

$$\ni g(x) = 0, \text{ i.e. } f(x) - x = 0, \text{ i.e. } f(x) = x$$

綜合 (情況一) (情況二) 得證

5. (B)

理由：(舉例說明)

例如：

$$f(x) = x^4 \quad f'(x) = 4x^3$$

$$g(x) = x^2 - 2x \quad g'(x) = 2x - 2$$

$$\lim_{x \rightarrow 1} \frac{f'(x)}{g'(x)} = \lim_{x \rightarrow 1} \frac{4x^3}{2x - 2} \rightarrow \text{不存在}$$

$$\text{但 } \lim_{x \rightarrow 1} \frac{f(x)}{g(x)} = \lim_{x \rightarrow 1} \frac{x^4}{x^2 - 2x} = -1$$

6. (A)

理由：(證明如下)

$$\text{令 } f(x) = \tan^{-1} x$$

$f(x)$  e cont. on  $[a, b]$

$$f'(x) = \frac{1}{1+x^2} \text{ e Def. on } [a, b] \quad b > a > 0$$

由 MVT 得： $\exists c (a < c < b)$

$$\ni : f'(c) = \frac{f(b) - f(a)}{b - a} = \frac{\tan^{-1} b - \tan^{-1} a}{b - a}$$

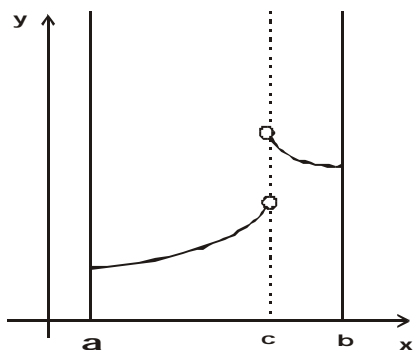
而  $f'(x)$  e  $\downarrow$  on  $[a, b]$

$$\therefore f'(b) < f'(c) < f'(a)$$

$$\text{i.e. } \frac{1}{1+b^2} < \frac{\tan^{-1} b - \tan^{-1} a}{b - a} < \frac{1}{1+a^2}$$

$$\therefore \frac{b-a}{1+b^2} < \tan^{-1} b - \tan^{-1} a < \frac{b-a}{1+a^2}$$

7. (B)



理由：

考慮函數  $f(x)$  如圖示

顯然  $f(x)$  為分段式連續

(*piecwisely continuous*)

$\therefore f(x)$  **e** integrable on  $[a, b]$

而  $F(x) = \int_a^x f(t)dt$ ,  $F'(c)$  不存在 ( $\because f(c)$  不存在)

$\therefore F(x) \notin$  differentiable at  $x = c$

$\therefore F(x) \notin$  differentiable on  $(a, b)$

8. (A)

理由：(證明如下)

$$\begin{aligned} F(3) &= \int_1^3 t^2 f(t) dt \leq \int_1^3 t^2 \cdot 3 dt \quad (\because f(t) \leq 3 \text{ 且 } t^2 \geq 0) \\ &= t^3 \Big|_1^3 = 26 \end{aligned}$$

9. (B)

理由：(舉例如下)

例如：

$$f(x, y) = \begin{cases} \frac{xy}{x^2 + y^2} & (x, y) \neq (0, 0) \\ 0 & (x, y) = (0, 0) \end{cases}$$

顯然  $f_x(0, 0) = 0, f_y(0, 0) = 0$

但是  $f(x, y)$  在  $(0, 0)$  不可微

( $\because \lim_{(x, y) \rightarrow (0, 0)} f(x, y)$  不存在  $\Rightarrow f(x, y)$  在  $(0, 0)$  不連續)

10. (B)

理由：(舉例如下)

例如函數  $f(x)$  如圖示：

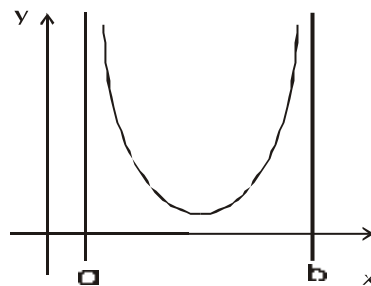
$f(x)$  **e** cont. on  $(a, b)$

且  $f(x)$  在  $x = a$ , 及  $x = b$  呈

無窮不連續

顯然無法找到  $x_0 \in (a, b)$

使得  $f(x) \leq f(x_0)$



11. (B)

$$\int_{-1}^3 \frac{6x-7}{3x+5} dx = \int_{-1}^3 \left(2 - \frac{17}{3x+5}\right) dx = \left[2x - \frac{17}{3} \ln(3x+5)\right]_{-1}^3 = 8 - \frac{17}{3} \ln 7$$

12.(A)

$$A = \int_0^1 x\sqrt{3x+1} dx$$

$$\text{令 } \sqrt{3x+1} = y \Rightarrow 3x+1 = y^2, 3x = y^2 - 1, x = \frac{1}{3}(y^2 - 1), dx = \frac{2}{3}y dy$$

$$\therefore A = \int_{\frac{1}{3}}^2 \frac{1}{3}(y^2 - 1) \cdot y \cdot \frac{2}{3}y dy = \frac{2}{9} \int_1^2 (y^4 - y^2) dy = \frac{2}{9} \left( \frac{1}{5}y^5 - \frac{1}{3}y^3 \right) \Big|_1^2 = \frac{2}{9} \cdot \frac{58}{15} = \frac{116}{135}$$

13.(C)

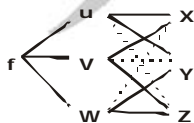
$$\text{依題可得微方 } \begin{cases} yy' + 2x = 0 \dots (1) \\ y(3) = -1 \dots (2) \end{cases}$$

$$(1) yy' = -2x \Rightarrow y \frac{dy}{dx} = -2x \Rightarrow y dy = -2x dx$$

$$\Rightarrow \text{通解: } \frac{1}{2}y^2 = -x^2 + C, \text{ 再將 } y(3) \text{ 帶入通解中, 可得 } C = \frac{19}{2}$$

$$\therefore \text{所求特解: } \frac{1}{2}y^2 = -x^2 + \frac{19}{2}, \text{ i.e. } y^2 + 2x^2 = 19$$

14.(B)



$$\begin{aligned} \frac{\partial f}{\partial x} + \frac{\partial f}{\partial y} + \frac{\partial f}{\partial z} &= \frac{\partial f}{\partial u} \cdot \frac{\partial u}{\partial x} + \frac{\partial f}{\partial v} \cdot \frac{\partial v}{\partial x} + \frac{\partial f}{\partial w} \cdot \frac{\partial w}{\partial x} + \frac{\partial f}{\partial u} \cdot \frac{\partial u}{\partial y} + \frac{\partial f}{\partial v} \cdot \frac{\partial v}{\partial y} + \frac{\partial f}{\partial w} \cdot \frac{\partial w}{\partial y} + \frac{\partial f}{\partial u} \cdot \frac{\partial u}{\partial z} \\ &\quad + \frac{\partial f}{\partial v} \cdot \frac{\partial v}{\partial z} + \frac{\partial f}{\partial w} \cdot \frac{\partial w}{\partial z} \\ &= \frac{\partial f}{\partial u} \cdot 1 + \frac{\partial f}{\partial v} \cdot 0 + \frac{\partial f}{\partial w} \cdot (-1) + \frac{\partial f}{\partial u} \cdot (-1) + \frac{\partial f}{\partial v} \cdot 1 + \frac{\partial f}{\partial w} \cdot 0 + \frac{\partial f}{\partial u} \cdot 0 + \frac{\partial f}{\partial v} \cdot (-1) + \frac{\partial f}{\partial w} \cdot 1 = 0 \end{aligned}$$

15.(C)

$$PQ = \left[ -\frac{3}{2}, 2 \right], \hat{e}_{PQ} = \frac{PQ}{\|PQ\|} = \frac{\left[ -\frac{3}{2}, 2 \right]}{\sqrt{\left(\frac{3}{2}\right)^2 + 2^2}} = \frac{\left[ -\frac{3}{2}, 2 \right]}{\frac{5}{2}} = \left[ -\frac{3}{5}, \frac{4}{5} \right]$$

$$\nabla f(2, 0) = [f_x(2, 0), f_y(2, 0)] = [1, 2]$$

$$\therefore D_{\hat{e}} f(2, 0) = \nabla f(2, 0) \cdot \hat{e}_{PQ} = [1, 2] \cdot \left[ -\frac{3}{5}, \frac{4}{5} \right] = 1$$

16.(D)

$$\begin{aligned} \ln f(x) &= \frac{1}{3} [4 \ln|x+1| + 2 \ln|x-5| - \ln|x-1|] \\ \Rightarrow \frac{1}{f(x)} f'(x) &= \frac{1}{3} \left[ 4 \cdot \frac{1}{x+1} + 2 \cdot \frac{1}{x-5} - \frac{1}{x-1} \right] \\ \therefore f'(x) &= \frac{1}{3} f(x) \left[ 4 \cdot \frac{1}{x+1} + 2 \cdot \frac{1}{x-5} - \frac{1}{x-1} \right] \\ f'(2) &= \frac{1}{3} f(2) \left[ 4 \cdot \frac{1}{2+1} + 2 \cdot \frac{1}{2-5} - \frac{1}{2-1} \right] = \frac{1}{3} \cdot 9 \cdot \left[ \frac{4}{3} - \frac{2}{3} - 1 \right] = -1 \end{aligned}$$

17.(B)

$$\lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{1}{an + bk} = \lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{1}{n} \cdot \frac{1}{a + b \frac{k}{n}} = \int_0^1 \frac{1}{a + bx} dx = \frac{1}{b} \cdot \ln(a + bx) \Big|_0^1 = \frac{1}{b} \ln \frac{a+b}{a}$$

18.(C)

$$\begin{aligned} \int_0^{\frac{p}{3}} |\sin x - \cos x| dx &= \int_0^{\frac{p}{4}} |\sin x - \cos x| dx + \int_{\frac{p}{4}}^{\frac{p}{3}} |\sin x - \cos x| dx \\ &= \int_0^{\frac{p}{4}} (\cos x - \sin x) dx + \int_{\frac{p}{4}}^{\frac{p}{3}} (\sin x - \cos x) dx = (\sin x - \cos x) \Big|_0^{\frac{p}{4}} + (-\cos x - \sin x) \Big|_{\frac{p}{4}}^{\frac{p}{3}} \\ &= \left[ \left( \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} \right) - (0 + 1) \right] + \left[ \left( -\frac{1}{2} - \frac{\sqrt{3}}{2} \right) - \left( -\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} \right) \right] = 2\sqrt{2} - \frac{3 + \sqrt{3}}{2} \end{aligned}$$

19.(B)

$$f(x) = xe^{-x} = x \sum_{n=0}^{\infty} \frac{1}{n!} (-x)^n = \sum_{n=0}^{\infty} \frac{1}{n!} (-1)^n x^{n+1} = \sum_{n=0}^{\infty} a_n x^n$$

$$x^4 \text{ 係數: } a_4 = \frac{1}{3!} (-1)^3 \Rightarrow a_4 = -\frac{1}{3!}$$

20.(C)

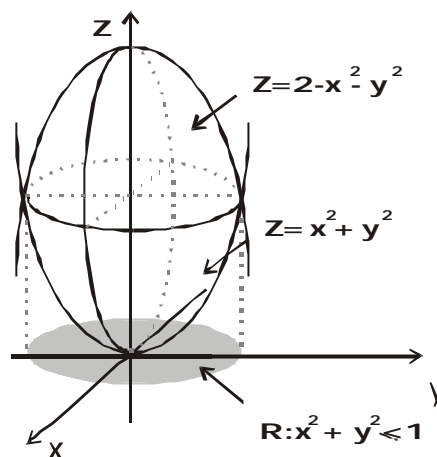
$$m(\mathbf{q}) = \frac{f'(\mathbf{q}) \sin \mathbf{q} + f(\mathbf{q}) \cos \mathbf{q}}{f'(\mathbf{q}) \cos \mathbf{q} - f(\mathbf{q}) \sin \mathbf{q}}, \text{ 其中 } f(\mathbf{q}) = \frac{\sqrt{2}}{2} + \cos \mathbf{q}, f'(\mathbf{q}) = -\sin \mathbf{q}$$

$$\therefore m\left(\frac{\mathbf{p}}{4}\right) = \frac{f'\left(\frac{\mathbf{p}}{4}\right) \sin \frac{\mathbf{p}}{4} + f\left(\frac{\mathbf{p}}{4}\right) \cos \frac{\mathbf{p}}{4}}{f'\left(\frac{\mathbf{p}}{4}\right) \cos \frac{\mathbf{p}}{4} - f\left(\frac{\mathbf{p}}{4}\right) \sin \frac{\mathbf{p}}{4}} = \frac{\left(-\frac{\sqrt{2}}{2}\right) \cdot \left(\frac{\sqrt{2}}{2}\right) + \left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\right) \cdot \frac{\sqrt{2}}{2}}{\left(-\frac{\sqrt{2}}{2}\right) \cdot \left(\frac{\sqrt{2}}{2}\right) - \left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\right) \cdot \frac{\sqrt{2}}{2}} = -\frac{1}{3}$$



21.(E)

$$\begin{aligned}
 V &= \iint_R [(2-x^2-y^2)-(x^2+y^2)] dx dy \\
 &= \int_0^{2p} \left[ \int_0^1 2 \cdot (1-r^2) r dr \right] d\mathbf{q} \\
 &= \left[ \int_0^{2p} d\mathbf{q} \right] \cdot \left[ \int_0^1 2(1-r^2) r dr \right] \\
 &= 2\mathbf{p} \cdot \frac{1}{2} = \mathbf{p}
 \end{aligned}$$



22.(D)

$$\frac{\sin 2x}{x} \leq f(x) \leq \frac{e^{2x}-1}{x} \quad 0 < x < 0.5$$

$$\begin{aligned}
 \lim_{x \rightarrow 0^+} \frac{\sin 2x}{x} &= \lim_{x \rightarrow 0^+} \frac{2 \cos 2x}{1} = 2, \quad \lim_{x \rightarrow 0^+} \frac{e^{2x}-1}{x} = \lim_{x \rightarrow 0^+} \frac{2e^{2x}}{1} = 2 \\
 \therefore \lim_{x \rightarrow 0^+} f(x) &= 2 \text{ (根據夾擊原理)} \quad \therefore \lim_{x \rightarrow 0^+} (2f(x))^{f(x)} = (2 \cdot 2)^2 = 16
 \end{aligned}$$

23.(C)

$$f(x) = \int_0^x (t-t^3)e^t dt \quad \mathbf{e} \text{ cont. on } (-\infty, \infty)$$

$$f'(x) = (x-x^3)e^x = -e^x(x+1)x(x-1)$$

$$\text{臨界數} : -1, 0, 1 \quad (f'(x) = 0)$$

$x$	$(-\infty, -1)$	$-1$	$(-1, 0)$	$0$	$(0, 1)$	$1$	$(1, \infty)$
$f'(x)$	$+$	$0$	$-$	$0$	$+$	$0$	$-$
	$\nearrow$		$\searrow$		$\nearrow$		$\searrow$

$$\text{相對極大} \quad f(-1) = \int_0^{-1} (t-t^3)e^t dt = e^t(-t^3+3t^2-5t+5) \Big|_0^{-1} = \frac{14}{e} - 5$$

$$\text{相對極大} \quad f(1) = \int_0^1 (t-t^3)e^t dt = e^t(-t^3+3t^2-5t+5) \Big|_0^1 = 2e - 5$$

$$\text{相對極小} \quad f(0) = \int_0^0 (t-t^3)e^t dt = 0$$

$$\text{而 } 2e - 5 > \frac{14}{e} - 5, \quad \therefore \text{得絕對極大 } f(1)$$

24.(D)

$$(\cos t)^4 > 0, \quad \forall t \quad \therefore \int_0^x (\cos t)^4 dt \leq 0 \quad \forall x < 0$$

25.(C)

$$\begin{aligned} \int_D \int x \cos xy dA &= \int_0^{\frac{p}{2}} x \left[ \int_1^2 \cos xy dy \right] dx = \int_0^{\frac{p}{2}} x \cdot \left[ \frac{1}{x} \sin xy \right]_1^2 dx = \int_0^{\frac{p}{2}} (\sin 2x - \sin x) dx \\ &= -\frac{1}{2} \cos 2x + \cos x \Big|_0^{\frac{p}{2}} = \left[ -\frac{1}{2}(-1) + 0 \right] - \left[ -\frac{1}{2} + 1 \right] = 0 \end{aligned}$$

26.(B)

$$\begin{aligned} \int_1^e x (\ln x)^2 dx &= \frac{1}{4} x^2 (2(\ln x)^2 - 2 \ln x + 1) \Big|_1^e \\ &= \frac{1}{4} e^2 (2 \cdot 1^2 - 2 \cdot 1 + 1) - \frac{1}{4} \cdot 1^2 \cdot (2 \cdot 0^2 - 2 \cdot 0 + 1) = \frac{e^2}{4} - \frac{1}{4} = \frac{1}{4} (e^2 - 1) \end{aligned}$$

建 國