高雄醫學大學九十三學年度學士後醫學系招生考試試題

科目:微積分　　　　　　　考試時間: 80 分鐘　　　　　　　　共 3 頁

說明:一、請用 2B 鉛筆在「答案卡」上作答，修正時應以橡皮擦拭，切勿使用修正液（帶），未遵照正確作答方法而致無法判讀者，考生自行負責。
二、試題及答案卡必須繳回，不得攜出試場。

（一）是非題：20 ％。（是，請在答案卡（A）欄劃記；非，請在答案卡（B）欄劃記。在其他欄位劃記者，不予計分。每題 2 分，答錯不倒扣。）

1. Let f be a continuous function such that $\lim\limits_{x\to\infty}\frac{f(x)}{x}=2$, then the infinite series $\sum_{n=1}^{\infty}\frac{f(n)}{n^3}$ converges.

2. Let f be a continuous function defined on interval $[a,b]$. If $f(a)f(b)-f(a)-f(b)+1<0$, then there exists a number c between a and b such that $f(c)=1$.

3. If f is integrable on $[a,b]$, then there exists a number $c\in[a,b]$ such that $\int_a^b f(x)dx=f(c)(b-a)$.

4. If $|f|$ is a Riemann integrable function on the interval [0, 1], then f is integrable on [0, 1].

5. Let f be a function defined as follows:
$$f(x)=\begin{cases}\sin x,\ x\in Q,\\ x,\ x\in\Re\setminus Q.\end{cases}$$
Then $f(x)$ is differentiable at $x = 0$.

6. We already know that the series $\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k}$ is convergent. Now, rewrite the series by combining 2^n consecutively positive terms in the series, then followed by 2^n consecutively negative terms, where n = 1, 2, 3, …, to get a new series as follows:
$(1+\frac{1}{3})-(\frac{1}{2}+\frac{1}{4})+(\frac{1}{5}+\frac{1}{7}+\frac{1}{9}+\frac{1}{11})-(\frac{1}{6}+\frac{1}{8}+\frac{1}{10}+\frac{1}{12})+\ldots$, then the new series is still convergent.

7. If $\lim\limits_{x\to\infty}f'(x)=1$, then $\lim\limits_{x\to\infty}\frac{f(x)}{x}=1$.

8. If $\sum a_n$ is a converging series with a_n for any $n\in N$, then $\lim\limits_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right|<1$.

9. If $f:(a,b)\to R$ has a relative extremum at $c\in(a,b)$, then either $f'(c)=0$ or $f'(c)$ does not exist.

10. If f is continuously differentiable and $z=f(x-y)$, then $z_x+z_y=0$.

（二）選擇題：80 %（單選題，每題 5 分，答錯一題倒扣 1.25 分，倒扣至本大題零分為止，未做答不給分亦不扣分。）

11. Let $f(x)=x\sin x$, then $\lim\limits_{n\to\infty}\frac{1}{n}\sum_{k=1}^{n}f(\frac{\pi}{4}+\frac{k\pi}{2n})$ = ?

(A) 1　(B) $\frac{\pi}{2}$　(C) $\sqrt{2}$　(D) 2　(E) $\frac{\pi}{\sqrt{2}}$

12. $\int_1^e (x\ln x)^2dx=?$

(A) $\frac{e^3}{3}-\frac{e^2}{2}+\frac{1}{6}$　(B) $\frac{e^3}{3}$　(C) $\frac{1}{3}$　(D) $\frac{5e^3}{27}-\frac{2}{27}$　(E) $\frac{e^3}{3}-\frac{1}{3}$

背面有試題

13. Find the tangent of the curve $x^3 - x^2y^2 + y^3 = 5$ at point $(1, 2)$.
(A) $5x + 24y - 53 = 0$ (B) $5x - 24y + 43 = 0$ (C) $5x - 8y + 11 = 0$ (D) $5x + 8y - 21 = 0$ (E) $5x - 12y + 19 = 0$

14. Consider the function $f(x) = \begin{cases} -x^2 - x & \text{if } x < 0 \\ 4x^3 - 15x^2 + 12x & \text{if } x \geq 0 \end{cases}$. The absolute minimum value of the function f on the interval $[-\frac{1}{2}, 1]$ is:

(A) -4 (B) 0 (C) $\frac{1}{4}$ (D) 1 (E) $\frac{11}{4}$

15. Find a and b so that function $f(x) = ax^3 + bx^2 + 1$ will have a relative minimum value at a point inside the open interval (1, 3).
(A) $a = 1, b = 1$ (B) $a = 1, b = -1$ (C) $a = -1, b = 2$ (D) $a = 2, b = -4$ (E) $a = -1, b = 3$

16. Consider the sphere $x^2 + y^2 + z^2 - 2x = 0$ and plane $\sqrt{6}\,x + y + z = 0$. Find the angle between normal lines of these two surfaces at point (0, 0, 0).

(A) $\frac{\pi}{6}$ (B) $\frac{5\pi}{6}$ (C) $\frac{\pi}{3}$ (D) $\frac{2\pi}{3}$ (E) $\frac{\pi}{2}$

17. Let $f(x)$ be a function defined by $f(x) = k + 1, \frac{1}{2^{k+1}} < x \leq \frac{1}{2^k}, k = 0, 1, 2, \cdots$

Which of the following items is the value of the integral $\int_0^1 f(x)\,dx$?

(A) 1 (B) 2 (C) 3 (D) 4 (E) ∞

18. Let $g(x)$ be the inverse function of the function $f(x) = xe^x$, where $x \geq 0$, i.e., $g(f(x)) = x$, and $f(g(x)) = x$ for $x \geq 0$. Which of the following items is the value of the integral $\int_0^e g(x)\,dx$?

(A) e – 1 (B) 1 (C) e (D) 1 + ln 2 (E) $e^2 - 1$

19. Let $f(x)$ be a function defined by $f(x) = \frac{1}{1 - \sin x}, |x| < \frac{\pi}{2}$.
Let $p(x) = a_0 + a_1 x + a_2 x^2 + \ldots + a_n x^n + \ldots$ be the Maclaurin series of $f(x)$.
Which of the following items is the value of a_3?

(A) -1 (B) 0 (C) $\frac{1}{6}$ (D) $\frac{5}{6}$ (E) 1

20. Which of the following items is the value of the integral $\int_e^{e^2} \frac{1 + 2\ln x}{x \ln x}\,dx$?

(A) ln2 (B) 2 (C) ln $2e$ (D) 2 + ln2 (E) $2e^2$

21. Let $f(x) = [x]$ be the greatest integer function, where $[x]$ is the greatest integer less than or equal to x. Which of the following items is the value of the integral $\int_{-1}^{2} f(x^2 + 1)\,dx$?

(A) 3 (B) 6 (C) $8 - \sqrt{2} - \sqrt{3}$ (D) 8 (E) $8 + \sqrt{2} + \sqrt{3}$

22. Let $a_1 = 1$ and $a_{n+1} = \sqrt{2a_n}$ for $n \in N$. Which of the following items is the value of the limit $\lim_{n\to\infty} a_n$?

(A) 2 (B) 1 (C) $\frac{1}{2}$ (D) e (E) ln 2

23. Find the volume of the region E bounded by $z = x^2 + y^2$, $x^2 + y^2 = 4$ and $z = 0$.

(A) $\frac{4\pi}{3}$ (B) 2π (C) $\frac{8\pi}{3}$ (D) 4π (E) 8π

24. Find $\int_0^1 \int_0^1 y^3 e^{xy^2}\, dy\, dx$.

(A) $e-2$ (B) $e-1$ (C) $\frac{e}{2}$ (D) $\frac{e-1}{2}$ (E) $\frac{e-2}{2}$

25. Determine whether the series converges.

(A) $\sum \frac{1}{1+\ln n}$ (B) $\sum n\sin($ (C) $\sum \frac{\ln n}{\sqrt{n}}$ (D) $\sum \frac{n}{e^n}$ (E) $\sum$

26. Find the directional derivative of $F(x, y, z) = xy + 2xz - y^2 + z^2$ at the point (1, −2, 1) along the curve

$x = t$, $y = t - 3$, $z = t^2$ in the direction of increasing z.

(A) $\frac{13}{\sqrt{6}}$ (B) $\frac{13}{6}$ (C) 11 (D) 13 (E) $\frac{11}{\sqrt{6}}$

後醫 93 解答

是非題

1. (A). $\sum_{n=1}^{\infty}\frac{f(n)}{n^3}\sim\sum_{n=1}^{\infty}\frac{2}{n^2}$ conv.

2. (A). 因 $f\in C[a,b]$ 且

$$f(a)f(b)-f(a)-f(b)+1<0 \iff (f(a)-1)(f(b)-1)<0$$
$$\iff 1 \text{ 介於 } f(a),f(b) \text{ 之間}$$

$\therefore \exists c\in(a,b),\ f(c)=1$

3. (B). 須 $f\in C[a,b]$

4. (B). 反例如 $f(x)=\begin{cases}1, & x\in\mathbb{Q}\\ -1, & x\in\mathbb{R}\setminus\mathbb{Q}\end{cases}$, 則 $|f|\in R[0,1]$, 但 $f\notin R[0,1]$

5. (A). 由 $\frac{f(x)-f(0)}{x-0}=\begin{cases}\frac{\sin x}{x}, & x\in\mathbb{Q}\\ 1, & x\in\mathbb{R}\setminus\mathbb{Q}\end{cases}$ 得 $\lim_{x\to0}\frac{f(x)-f(0)}{x-0}=1$ $\therefore f$ 在 $x=0$ 可微

6. (B). 新級數的第 $2n$ 項爲

$$a_{2n}=-\frac{1}{2}\left(\frac{1}{(2^n-2)+1}+\frac{1}{(2^n-2)+2}+\frac{1}{(2^n-2)+3}+\cdots+\frac{1}{2^{n+1}-2}\right)$$

於是

$$|a_{2n}|\geq\frac{1}{2}\cdot\frac{2^n}{2^{n+1}-2}\geq\frac{1}{2}\cdot\frac{2^n}{2^{n+1}}=\frac{1}{4};\qquad \lim_{n\to\infty}|a_{2n}|\neq0$$

$\therefore$ 新級數 div.

7. (A). 由 Mean Value Theorem 可以証得

8. (B). 反例如 $a_n=\frac{1}{n^2}$, 有 $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right|=1$

9. (A). 這是定理

10. (A). 由 chain rule: $z_x=f'(x-y)\cdot(1)$; $z_y=f'(x-y)\cdot(-1)$ $\qquad\therefore z_x+z_y=0$

選擇題

11. $\lim_{n\to\infty}\frac{1}{n}\sum_{k=1}^{n} f\left(\frac{\pi}{4}+\frac{k\pi}{2n}\right)=\frac{2}{\pi}\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} f(x)\,dx=\frac{2}{\pi}\Big[-x\cos x+\sin x\Big]_{\frac{\pi}{4}}^{\frac{3\pi}{4}}=\frac{2}{\pi}\left[\frac{3\pi}{4\sqrt{2}}+\frac{\pi}{4\sqrt{2}}\right]=\sqrt{2}$

答 (C)

12. 令 $\ln x=t$, 則

$$\int_1^e (x\ln x)^2\,dx=\int_0^1 t^2e^{3t}\,dt=e^{3t}\left[\frac{1}{3}t^2-\frac{2}{9}t+\frac{2}{27}\right]_0^1=\frac{5e^3}{27}-\frac{2}{27}$$

答 (D)

13. $\boldsymbol{n}=(3x^2-2xy^2,-2x^2y+3y^2)\Big|_{(1,2)}=-(5,-8)$ 答 (C)

14. $f'(x)=\begin{cases}2x-1, & x<0\\ 6(2x-1)(x-2), & x>0\end{cases}$

x	$\frac{-1}{2}$	0		$\frac{1}{2}$	1
f'	$-$		$+$		$-$
f	↘	0	↗		↘ 1

$\therefore \min f=0$ 答 (B)

15. $f'(x)=3ax^2+2bx=x(3ax+2b)$, 須 $1<-\frac{2b}{3a}<3$ 答 (D)

16. $\boldsymbol{n}_1=(2x-2,2y,2z)\Big|_{(0,0,0)}=(-2,0,0),\ \ \boldsymbol{n}_2=(\sqrt{6},1,1)$

$$\cos\theta=\frac{\boldsymbol{n}_1\cdot\boldsymbol{n}_2}{\|\boldsymbol{n}_1\|\,\|\boldsymbol{n}_2\|}=\frac{-2\sqrt{6}}{(2)(\sqrt{8})}=-\frac{\sqrt{3}}{2};\qquad \theta=\frac{5\pi}{6}$$

答 (B)

17. $\int_0^1 f(x)\,dx=\sum_{k=0}^{\infty}(k+1)\left[\frac{1}{2^k}-\frac{1}{2^{k+1}}\right]=\sum_{k=0}^{\infty}\frac{k+1}{2^{k+1}}=\frac{x}{(1-x)^2}\Big|_{x=\frac{1}{2}}=2$ 答 (B)

18. $\int_0^e g(x)\,dx=x^2e^x\Big|_0^1-\int_0^1 xe^x\,dx=e-\Big[e^x(x-1)\Big]_0^1=e-1$ 答 (A)

19. $f(x)=\dfrac{1}{1-\left(x-\dfrac{x^3}{6}+o(x^3)\right)}$

$$=1+\left(x-\frac{x^3}{6}+o(x^3)\right)+\left(x-\frac{x^3}{6}+o(x^3)\right)^2+\left(x-\frac{x^3}{6}+o(x^3)\right)^3+o(x^3)$$

$$=1+x+x^2+\frac{5}{6}x^3+o(x^3)$$

答 (D)

20. 令 $\ln x=t$, 則 $\displaystyle\int_e^{e^2}\frac{1+2\ln x}{x\ln x}\,dx=\int_1^2\frac{1+2t}{t}\,dt=\ln t+2t\Big|_1^2=2+\ln 2$ 答 (D)

21. $\displaystyle\int_{-1}^2 f(x^2+1)\,dx=\int_{-1}^1 1\,dx+\int_1^{\sqrt{2}}2\,dx+\int_{\sqrt{2}}^{\sqrt{3}}3\,dx+\int_{\sqrt{3}}^2 4\,dx$

$$=2+2(\sqrt{2}-1)+3(\sqrt{3}-\sqrt{2})+4(2-\sqrt{3})=8-\sqrt{2}-\sqrt{3}$$

答 (C)

22. $a_n=2^{\frac{1}{2}+\frac{1}{2^2}+\cdots+\frac{1}{2^{n-1}}}=2^{1-\frac{1}{2^{n-1}}}\Rightarrow\displaystyle\lim_{n\to\infty}a_n=2^1=2$ 答 (A)

23. $V=\displaystyle\iint_{x^2+y^2\le 4}(x^2+y^2)\,dx\,dy=\int_0^{2\pi}\int_0^2 r^3\,dr\,d\theta=2\pi\cdot\frac{2^4-0}{4}=8\pi$ 答 (E)

24. $I=\displaystyle\int_0^1\int_0^1 y^3e^{xy^2}\,dx\,dy=\int_0^1 ye^{xy^2}\Big|_0^1 dy=\int_0^1 ye^{y^2}-y\,dy=\frac{1}{2}e^{y^2}-\frac{y^2}{2}\Big|_0^1=\frac{e-2}{2}$ 答 (E)

25. $\displaystyle\lim_{n\to\infty}\sqrt[n]{\frac{n}{e^n}}=\lim_{n\to\infty}\frac{\sqrt[n]{n}}{e}=\frac{1}{e}<1\Rightarrow\sum_{n=1}^{\infty}\frac{n}{e^n}$ conv. 答 (D)

26. $\nabla F(1,-2,1)=(y+2z,x-2y,2x+2z)\Big|_{(1,-2,1)}=(0,5,4)$; $\boldsymbol{v}=(1,1,2t)\Big|_{t=1}=(1,1,2)$

$\therefore D_{\boldsymbol{v}}F(1,-2,1)=(0,5,4)\cdot\dfrac{(1,1,2)}{\sqrt{6}}=\dfrac{13}{\sqrt{6}}$ 答 (A)